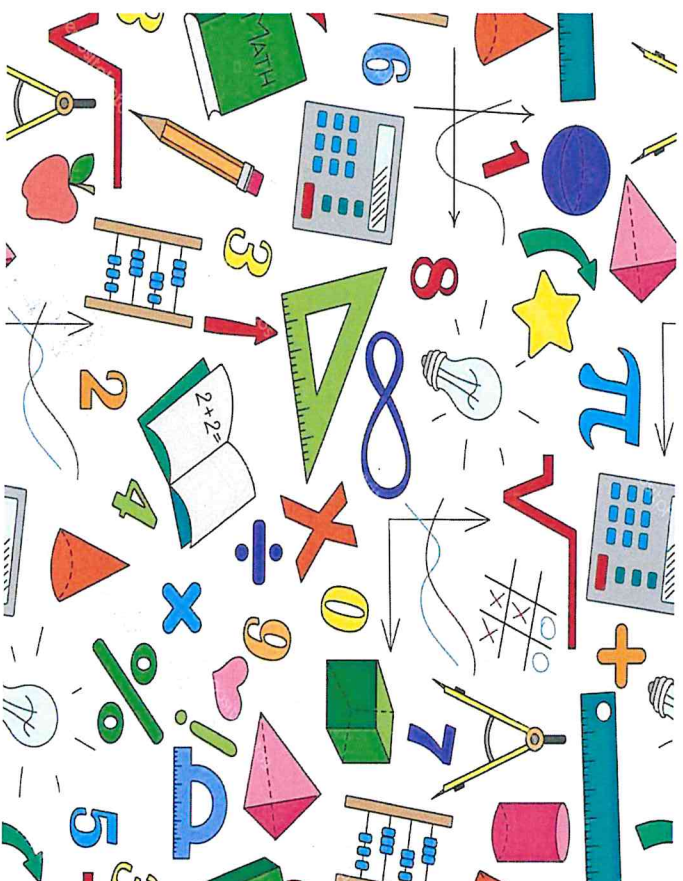
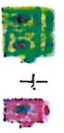


Endeavour
Schools Trust
Inspired to Excel

Endeavour Trust Calculation Policy



Endeavour
Schools Trust
Inspired to Excel



CONCRETE



PICTORIAL

$$2 + 1 = 3$$

ABSTRACT



CONCRETE

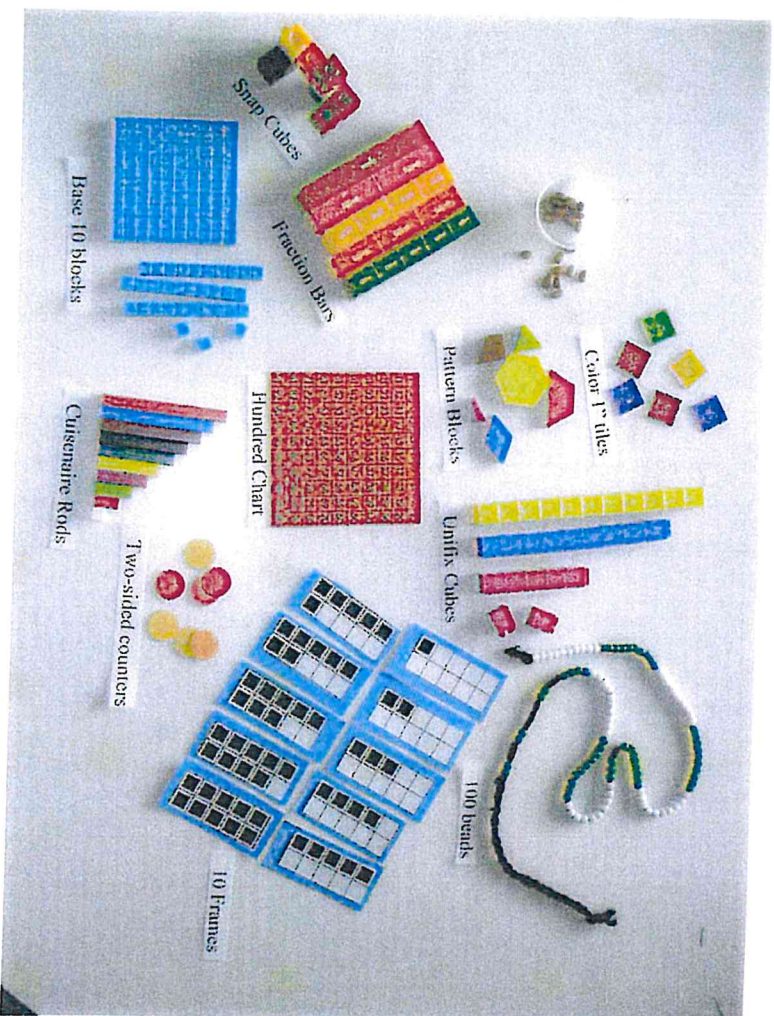


PICTORIAL

$$2 + 1 = 3$$

ABSTRACT

Concrete, Pictorial and Abstract Approach



Concrete:

The kinaesthetic 'doing' stage. Children use a variety of equipment to physically make and see the maths.



CONCRETE



PICTORIAL

$$2 + 1 = 3$$

ABSTRACT

Concrete, Pictorial and Abstract Approach



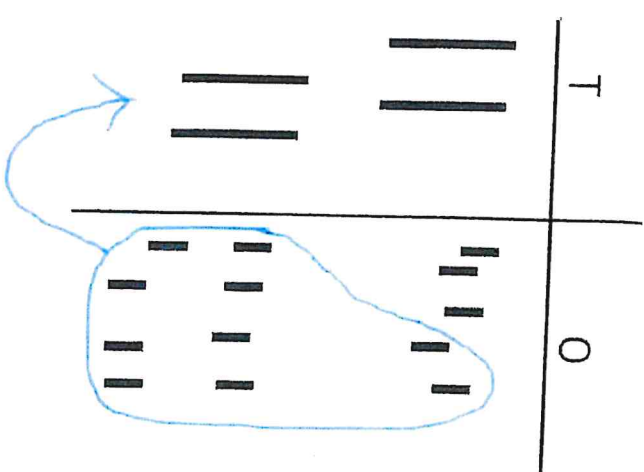
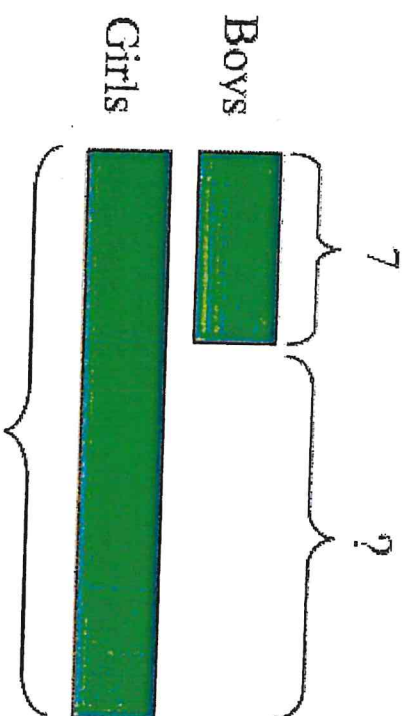
CONCRETE



PICTORIAL

$$2 + 1 = 3$$

ABSTRACT



$$\begin{array}{r} 25 \\ +18 \\ \hline 43 \\ 1 \end{array}$$

Pictorial:

The drawing 'seeing' stage. Children draw visual representations to help see and solve Mathematical problems.



CONCRETE

PICTORIAL

ABSTRACT

2 + 1 = 3

Concrete, Pictorial and Abstract Approach



CONCRETE

PICTORIAL

ABSTRACT

2 + 1 = 3

$$3 + 2 = \boxed{5}$$

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

137	r5
7	² ₅ 964

Abstract:

The symbolic stage. Children write sums, using mathematical symbols and formal methods to solve problems.

addition

how many more?

plus

counting on



total

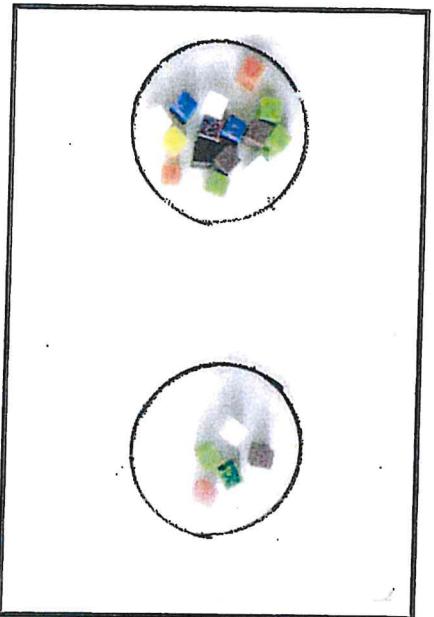
altogether

find the sum of...

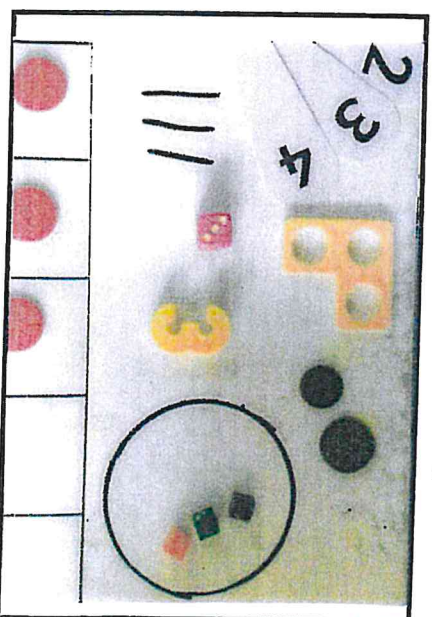
add

Stage 1 – Introducing numbers and learning how to count

Visually seeing more as a quantity (C)



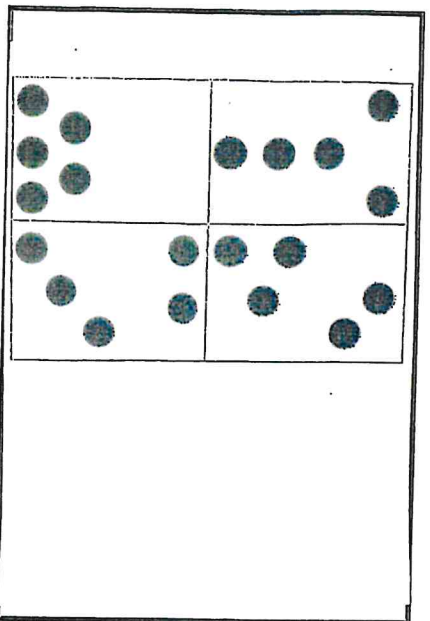
Introducing numbers in lots of contexts (C)



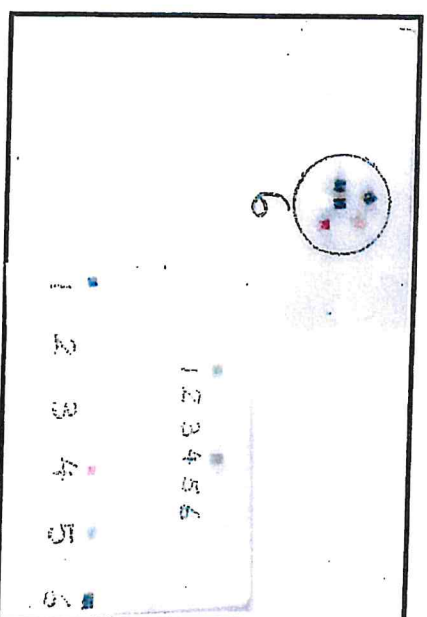
Vocabulary

- Number
- Counting
- Count on
- Which has more?

Subitising – ability to recognize numbers without the need to count (P)

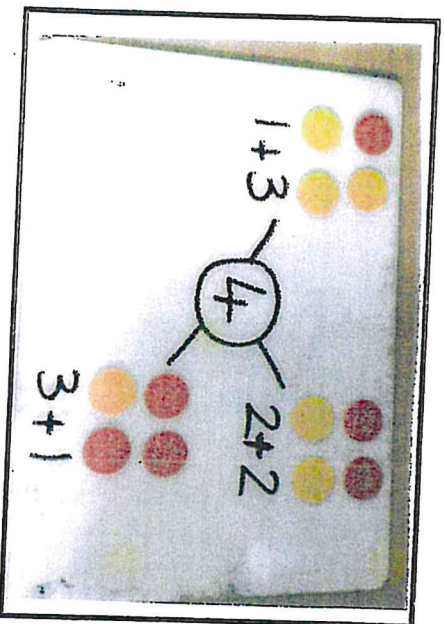


Counting objects and making amounts (C)

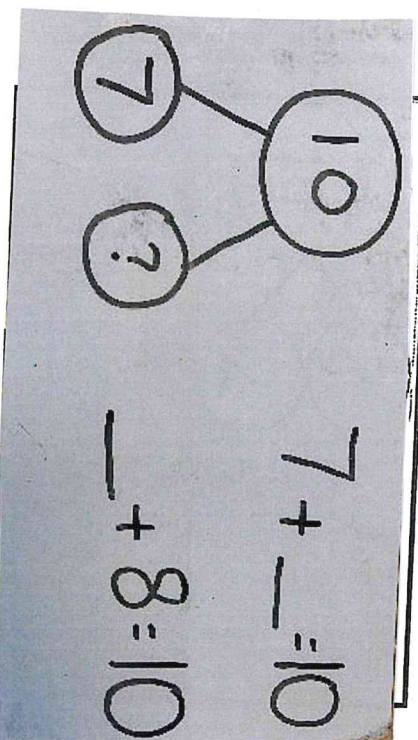


Stage 2 – Introduction of addition (Important to continue to develop mental +)

Number sense – finding all the ways to make numbers (C)



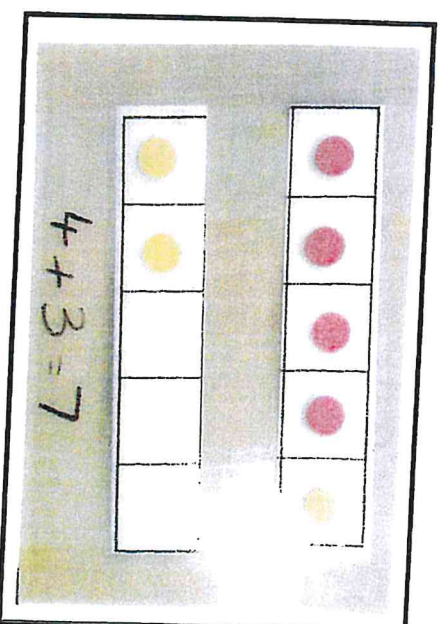
Number bonds to 10 (P / A)



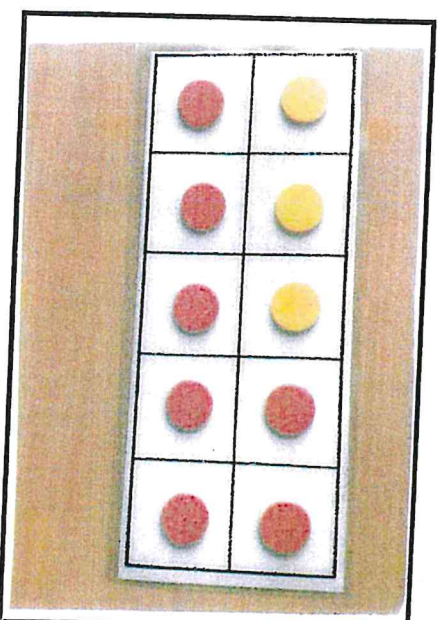
Vocabulary

- Bonds
- Pairs
- Count on
- How many more?
- Ones

Using 5 frames to add before 10 (C)



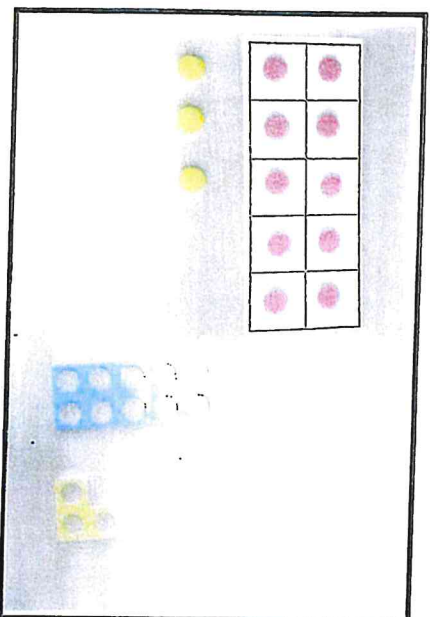
Number bonds to 10 (C)



Stage 3 – Bridging through 10 and working with ‘teen’ numbers

(Eventually, children need to be able to use their number bonds mentally or spot patterns in numbers to add proficiently).

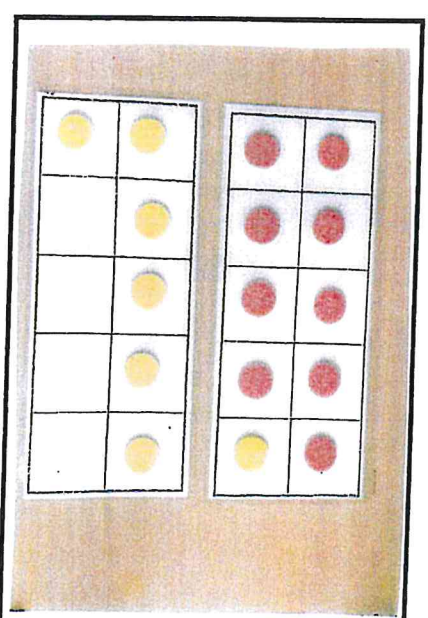
Creating 2 digit ‘teen’ numbers (C)



Vocabulary

- 2 digits
- Get to 10
- How many more?
- Add
- Plus

Bridging ten (C)



Bridging ten (P)

$$8 + 4 = 12$$

$$\begin{array}{c} \text{oo} \\ \text{oo} \\ \text{oo} \end{array} + \begin{array}{c} \text{oo} \\ \text{oo} \end{array}$$

or

$$8 + 4 = 12$$

“Get to the next 10!”

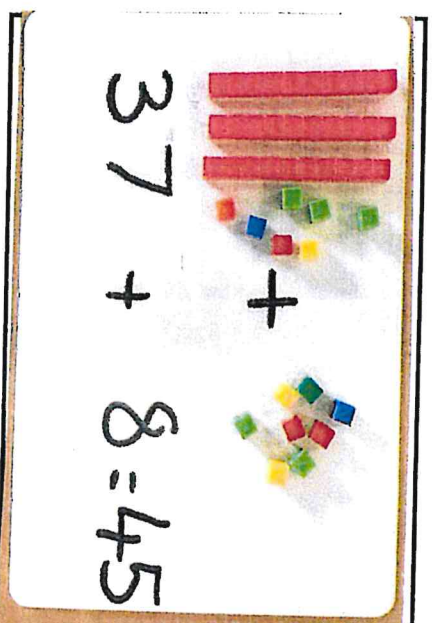
$$\begin{array}{c} \text{oo} \\ \text{oo} \\ \text{oo} \end{array} + \begin{array}{c} \text{oo} \\ \text{oo} \end{array}$$

$$3 + 4 = 7$$

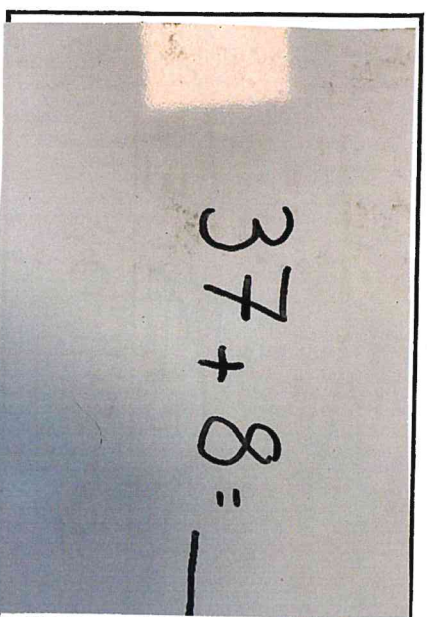


Stage 4 – 2 digit numbers + one digit number (37+8=)

2 digit numbers + single digit (C)



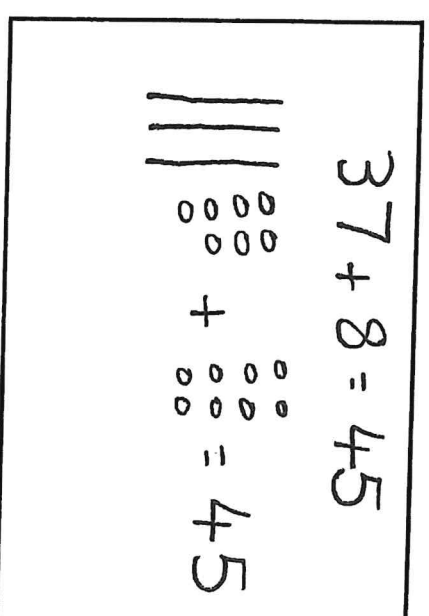
2 digit numbers + single digit (A)



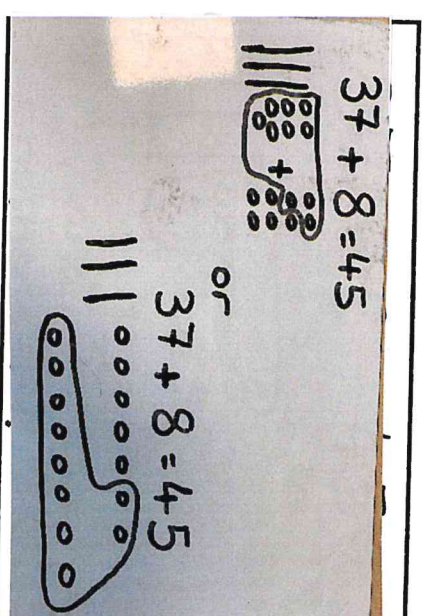
Vocabulary

- Get to 10
- How many more?
- Add
- Plus
- Altogether

2 digit numbers + single digit (P)

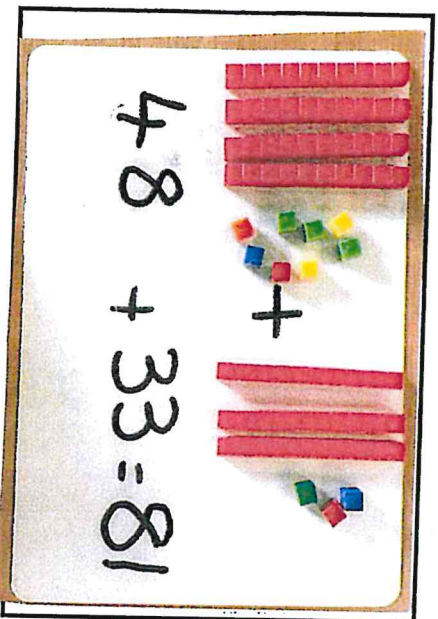


2 digit numbers + single digit (P)



Stage 5 – 2 digit numbers + 2 digit numbers (48 + 33=)

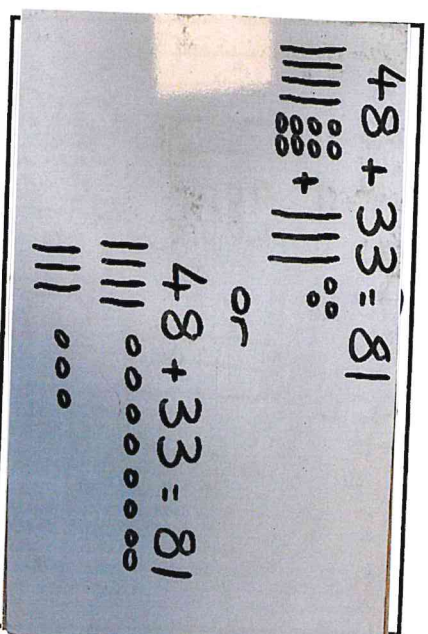
2 digit + 2 digit (C)



2 digit + 2 digit (A)

- Vocabulary**
- Get to 10
 - Add
 - Plus
 - Total
 - Altogether

2 digit + 2 digit (P)



2 digit + 2 digit (A)

48 + 33 = 81

Mental partitioning

Partitioning

48 + 33 =

40 + 8 30 + 3

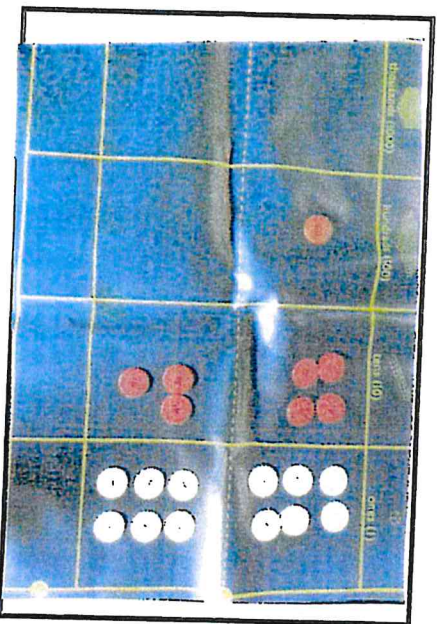
40 + 30 = 70

8 + 3 = 11

70 + 11 = 81

Stage 6 – 3 digit numbers + 2 digit numbers (146 + 36=)

3 digit + 2 digit (before) (C)



3 digit + 2 digit – expanded (A)

$$146 + 36 = 182$$

100	40	6
+	30	6
100	80	2
10		

Vocabulary

- Add
- Plus
- Sum of
- Total
- Altogether
- 3 digit
- Hundreds

3 digit + 2 digit (after) (C)



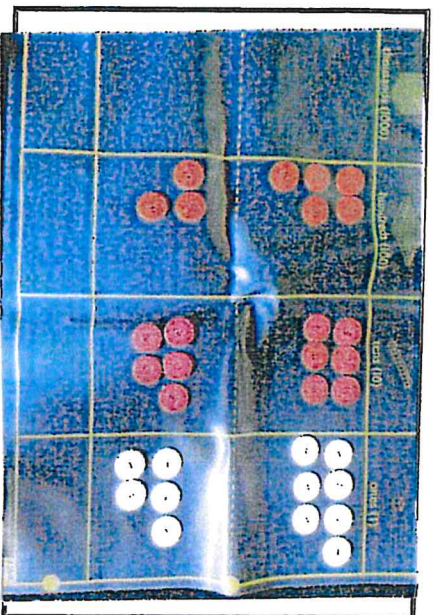
3 digit + 2 digit (P)

$$146 + 36 = 182$$

H	T	O
(H)	(T) (T) (T) (T) (T) (T)	(O) (O) (O) (O) (O) (O)
	(T) (T) (T) (T) (T) (T)	(O) (O) (O) (O) (O) (O)
1		2

Stage 7 – 3 digit numbers + 3 digit numbers (567 + 355 =)

3 digit + 3 digit (before) (C)



3 digit + 3 digit - compact (A)

$$567 + 355 = 922$$

$$\begin{array}{r} 567 \\ + 355 \\ \hline 922 \end{array}$$

3 digit + 3 digit (after) (C)



3 digit + 3 digit - expanded (A)

$$567 + 355 = 922$$

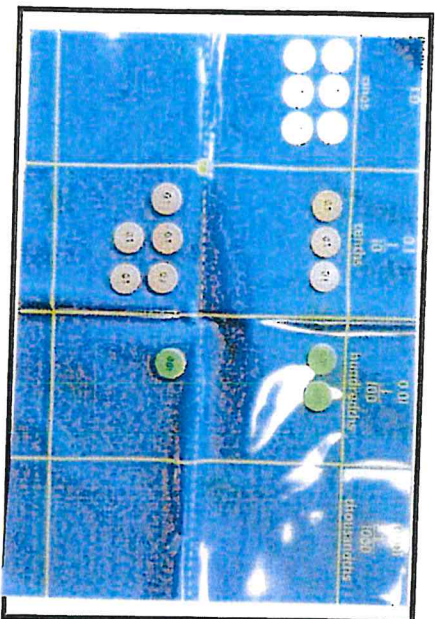
$$\begin{array}{r} 500 & 60 & 7 \\ + & 300 & 50 & 5 \\ \hline 900 & 20 & 2 \\ 100 & 10 & \end{array}$$

Vocabulary

- Add
- Plus
- Sum of
- Total
- Altogether
- 3 digit
- Hundreds

Stage 8 – Adding decimals (6.32 + 0.51 =)

Adding decimals (before) (C)



Adding decimals - compact (A)

$$\begin{array}{r} 6.32 \\ + 0.51 \\ \hline 6.83 \end{array}$$

Vocabulary

- Tenths
- Hundredths
- Decimals
- Parts of numbers

Adding decimals (after) (C)



Adding decimals (P)

$$\begin{array}{r} 6.32 \\ + 0.51 \\ \hline 6.83 \end{array}$$

subtract

what's the difference?

fewer

minus



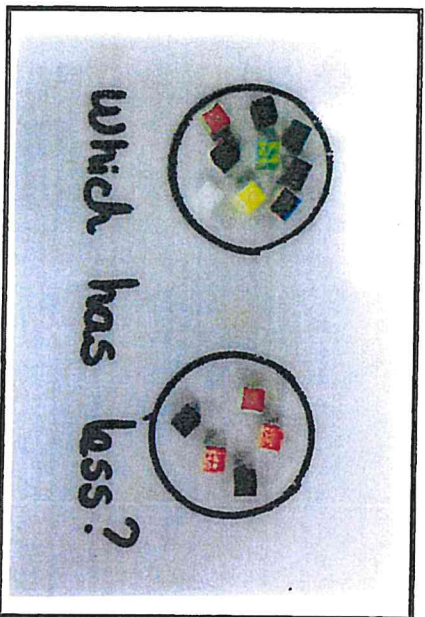
reduce

take away

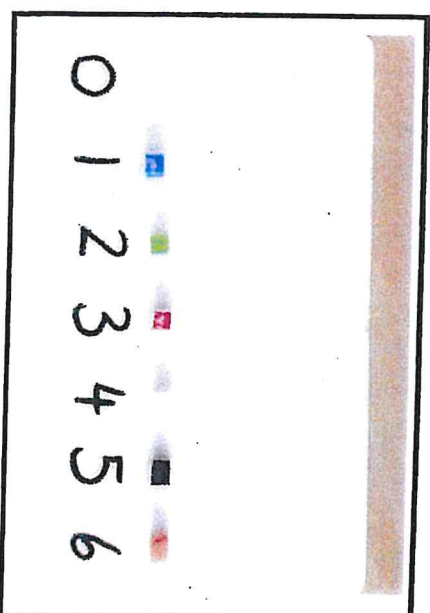
how many less?

Stage 1 – Identify less than another quantity and count backwards

Visually seeing less as a quantity (C)



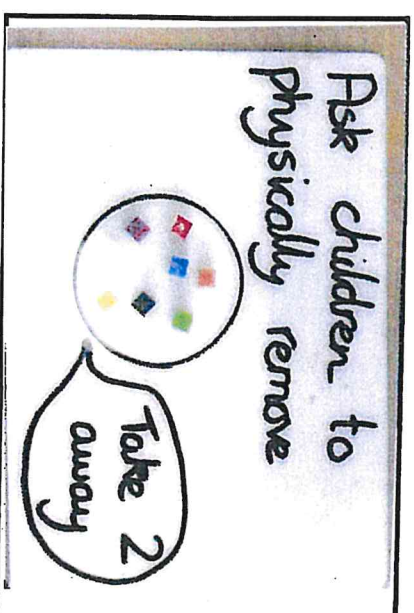
Counting backwards (including zero) (C)



Vocabulary

- number
- counting
- count back
- which has less?

Counting backwards (including zero) (C)

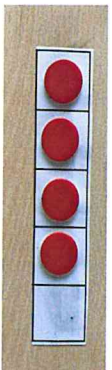


Stage 2 – Introduction of take away (Important to continue to develop mental -)

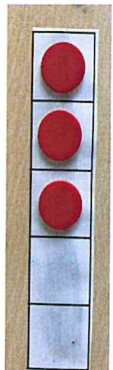
Using 5 frames to take away (C)

$4 - 1 =$

Before

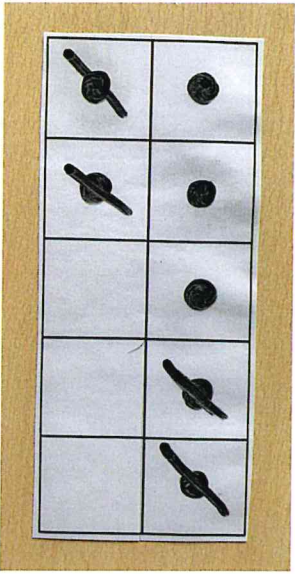


After



Using 10 frames to take away (P)

$7 - 4 =$



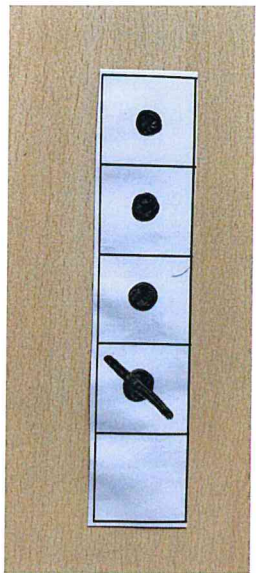

Vocabulary

- take away
- count back
- how many less?
- ones



Using 5 frames to take away (P)

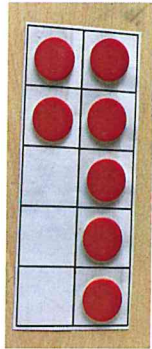
$4 - 1 =$



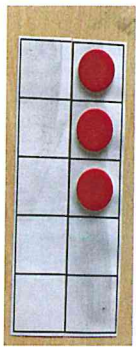
Using 10 frames to take away (C)

$7 - 4 =$

Before



After



Stage 3 – Bridging back through 10 (working within number to 20).

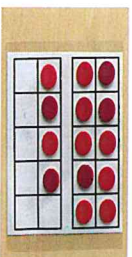
(Eventually, children need to be able to use their number bonds mentally or spot patterns in numbers to subtract proficiently).

Bridging 10 (C)

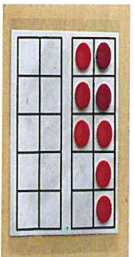
$$14 - 6 =$$

(How many do I subtract to get back to 10)?

Before



After



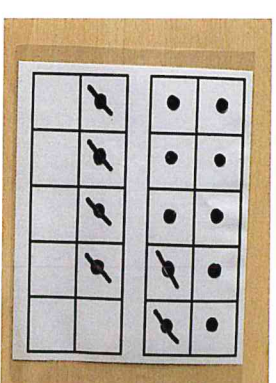
Vocabulary

- 2 digits
- get back to 10
- take away
- count back

Bridging ten (P)

$$14 - 6 =$$

(How many do I subtract to get back to 10)?



Using a number line

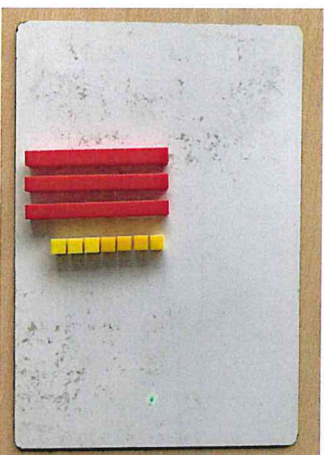
$$13 - 9 = 4$$



Stage 4 – 2 digit numbers – tens number (37 - 20=)

2 digit number – tens number (before) (C)

$$37 - 20 =$$

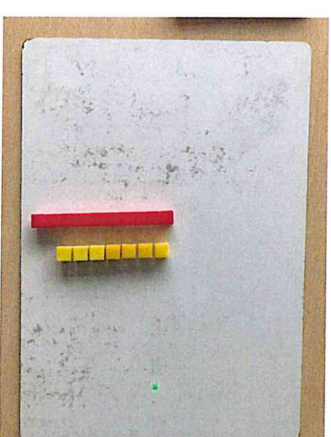


2 digit numbers – tens number (A)

$$\begin{array}{l} 37 - 20 = 17 \\ 53 - 10 = 43 \\ 86 - 40 = 46 \\ 79 - 30 = 49 \end{array}$$

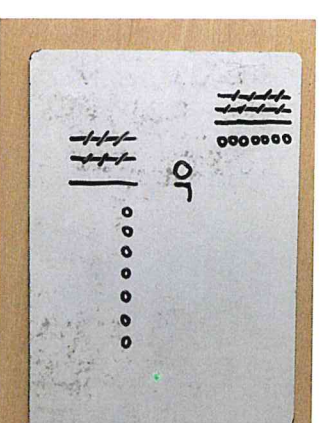
2 digit number – tens number (after) (C)

$$37 - 20 =$$



2 digit numbers – tens number (P)

$$37 - 20 =$$



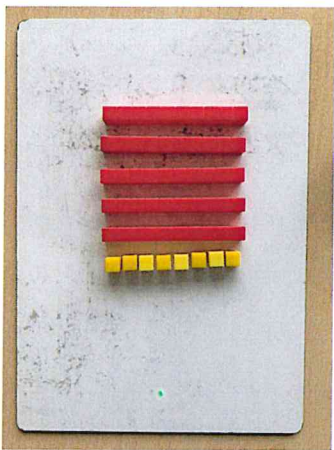
Vocabulary

- take away
- subtract
- tens
- 2 digit
- count back
- number line

Stage 5 – 2 digit numbers - 2 digit numbers - NO EXCHANGE (58 - 23=)

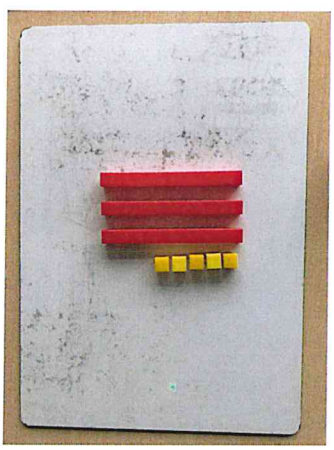
2 digit - 2 digit (before) (C)

$58 - 23 =$



2 digit - 2 digit (after) (C)

$58 - 23 =$



Vocabulary

- take away
- subtract
- find the difference
- tens
- ones
- number bonds

2 digit - 2 digit (A)

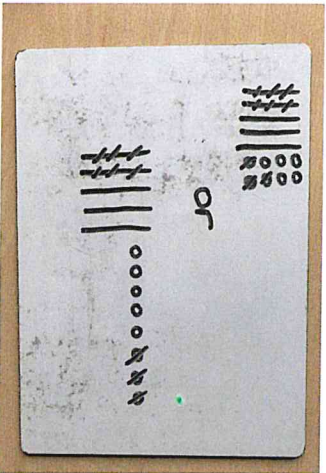
$58 - 23 =$

$50 - 20 = 30$

$8 - 3 = 5$

2 digit - 2 digit (P)

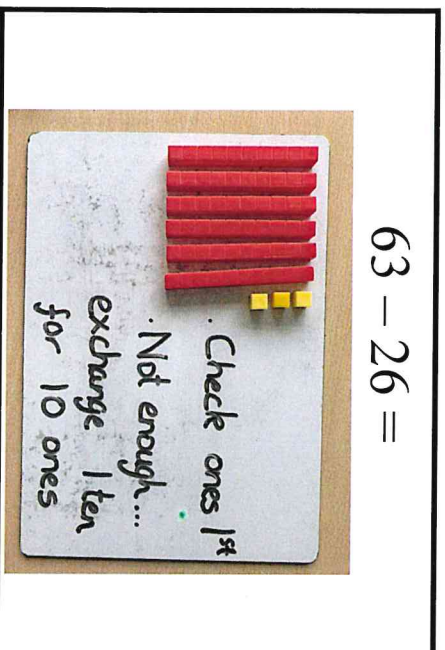
$58 - 23 =$



Stage 6 – 2 digit numbers - 2 digit numbers **EXCHANGE** (63 - 26=)

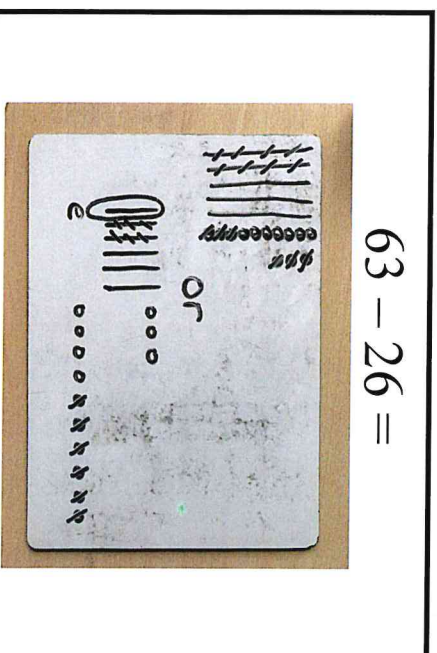
2 digit - 2 digit (part 1) (C)

$$63 - 26 =$$



2 digit - 2 digit (P)

$$63 - 26 =$$

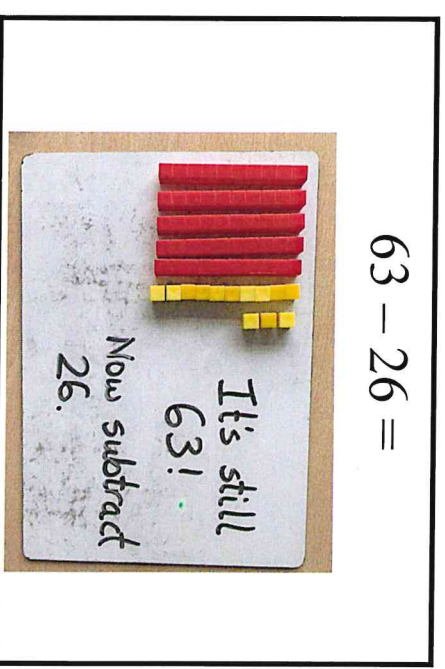


Vocabulary

- take away
- subtract
- find the difference
- exchange
- tens
- ones
- column

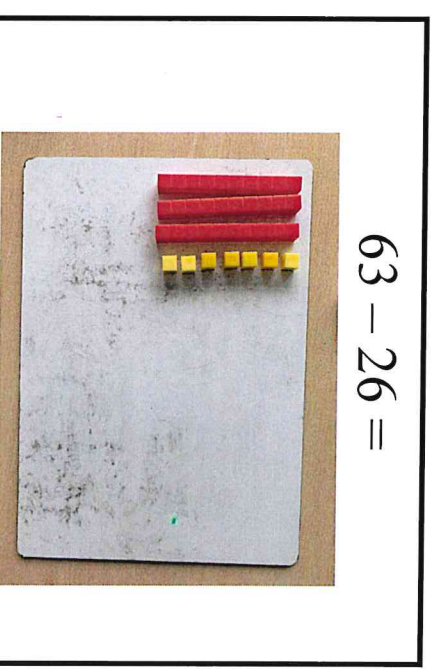
2 digit - 2 digit (part 2) (C)

$$63 - 26 =$$



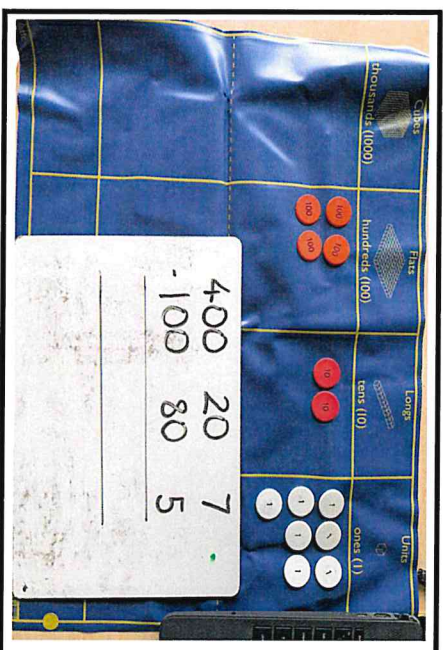
2 digit - 2 digit (part 3) (C)

$$63 - 26 =$$



Stage 7 – 3 digit numbers - 3 digit numbers – EXPANDED METHOD (427 - 185 =)

3 digit - 3 digit (C & A)



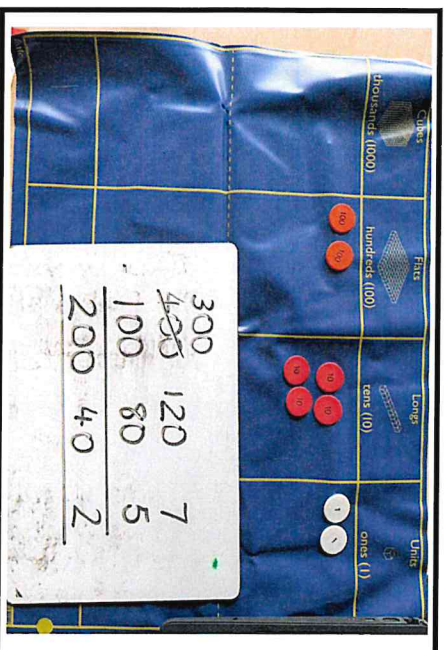
Vocabulary

- take away
- subtract
- find the difference
- exchange
- hundreds
- tens
- ones
- column

3 digit - 3 digit (C & A)



3 digit - 3 digit (C & A)



3 digit - 3 digit (C & A)

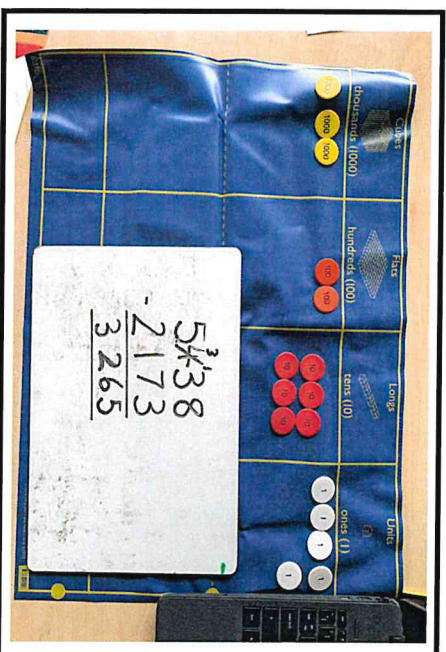


Stage 7b – 4 digit numbers – 4 digit numbers – COMPACT METHOD (5438 – 2173)

4 digit – 4 digit (C & A)



4 digit – 4 digit (C & A)



Vocabulary

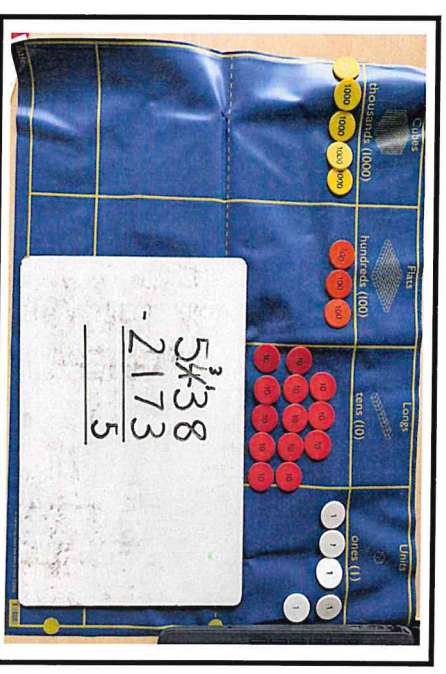
- take away
- subtract
- find the difference
- exchange
- hundreds
- tens
- ones
- column



4 digit – 4 digit (C & A)

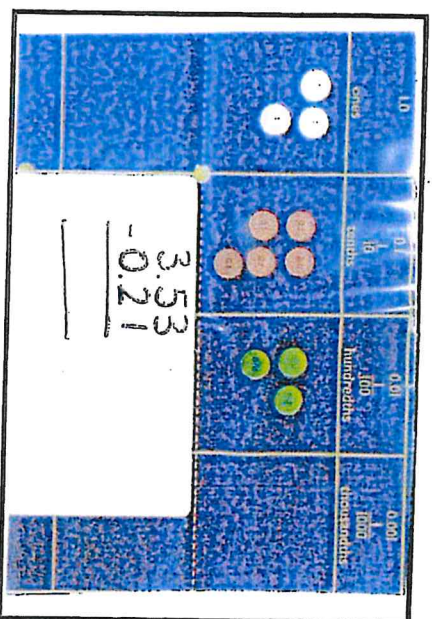


4 digit – 4 digit (C & A)

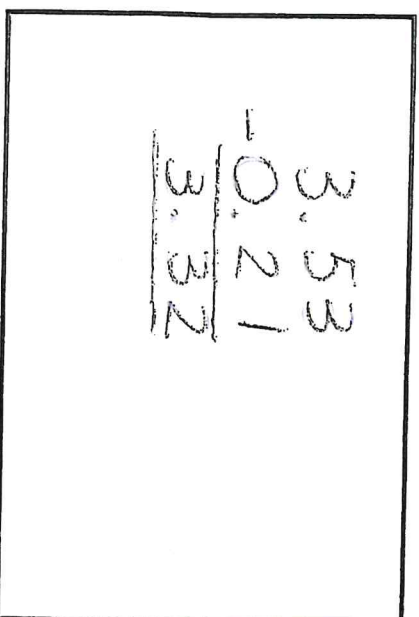


Stage 8 – Subtracting decimals

Subtracting decimals (step 1) (C)



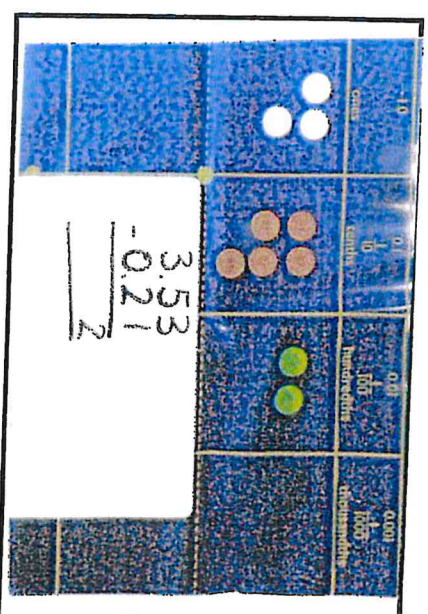
Subtracting decimals - compact (A)



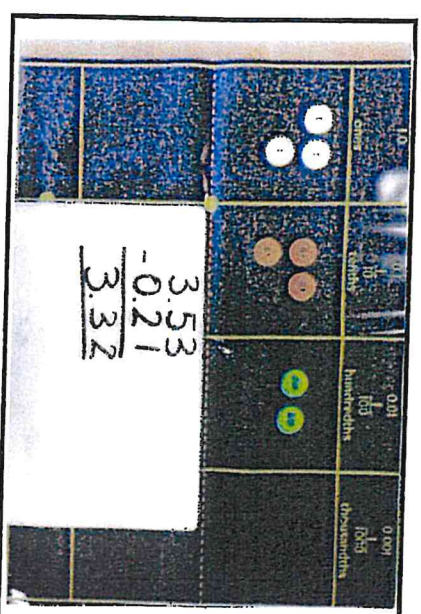
Vocabulary

- tenths
- hundredths
- decimals
- parts of numbers
- column

Subtracting decimals (step 2) (C)



Subtracting decimals (step 3) (C)

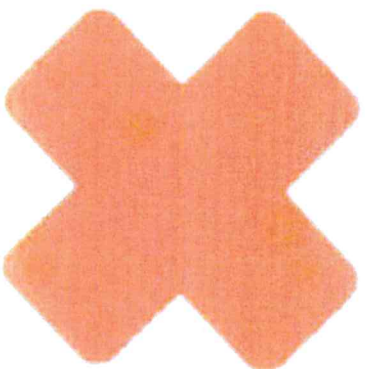


multiply

lots of

times

array



doubling

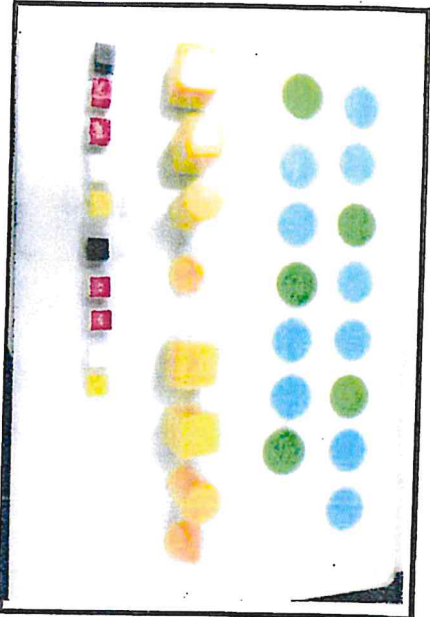
repeated ++

find the product of

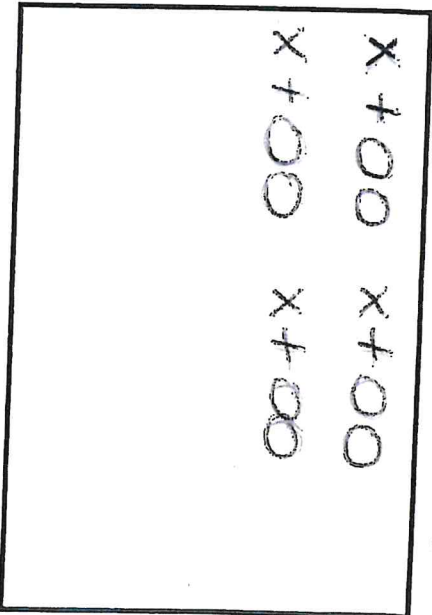
groups of

Stage 1 – Recognising and continuing repeated patterns

Visually seeing patterns as pictures (C)



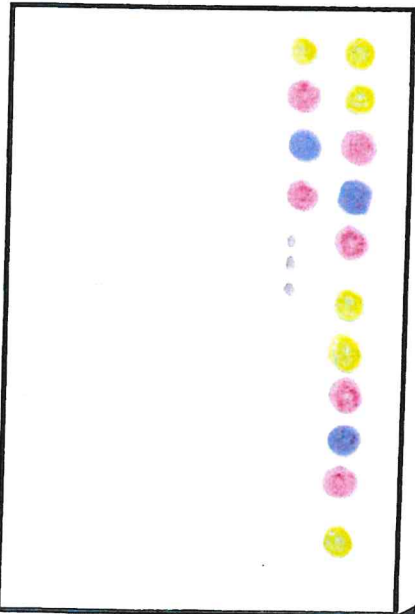
Visually seeing patterns as symbols (C)



Stage 1 targets:

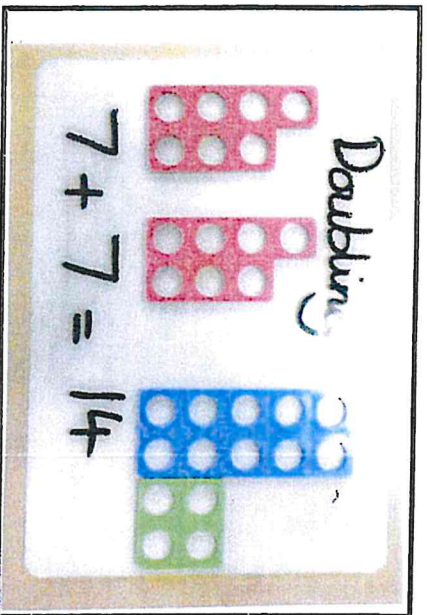
- Children recognise a variety of patterns (toys, colours, symbols, numbers, letters)
- Begin to continue these patterns
- Can children create and discuss their own?

Recognise and continue patterns(C)



Stage 2 – Doubling numbers to 10 in a range of contexts

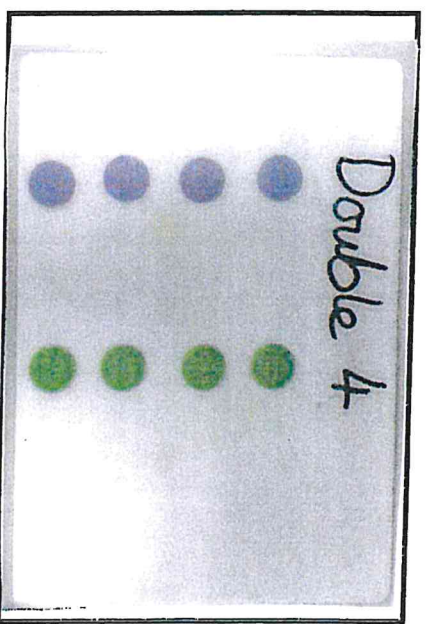
Doubling practically (C)



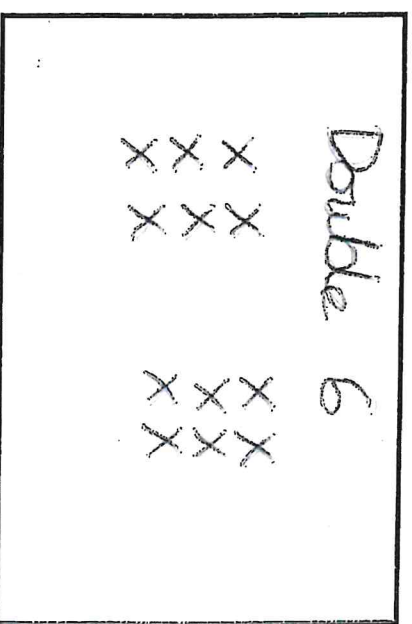
Vocabulary

- double
- 2 lots of
- repeat
- 2 times

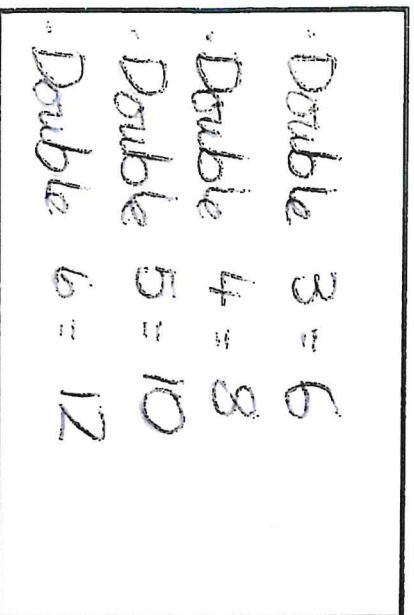
Doubling practically (C)



Doubling (P)



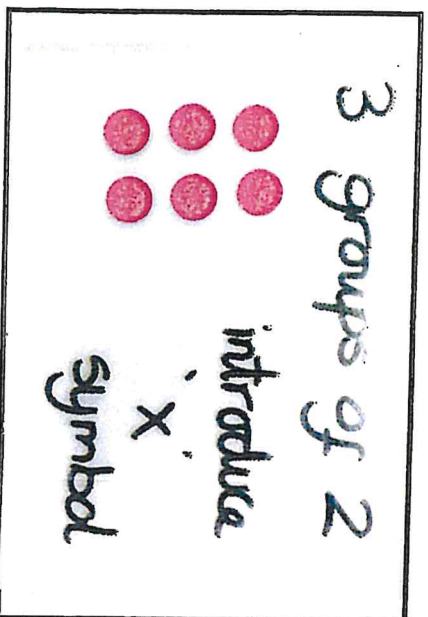
Doubling (A)



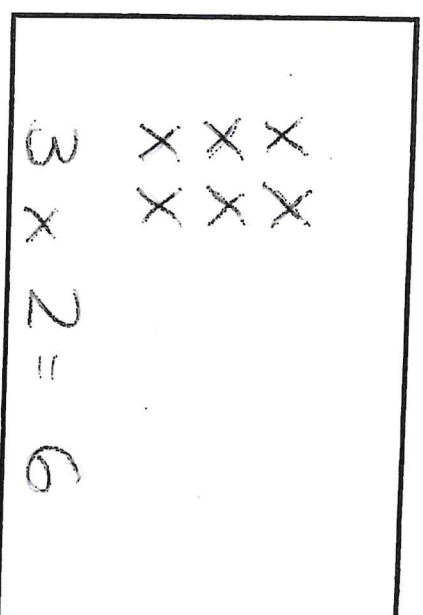
Stage 3 – Introducing multiplication symbol ($2\times$, $10\times$ and $5\times$).

(At this stage, lots of questions should be based on the above tables)

Multiplying by 2×10 and 5 – set out as array (C)



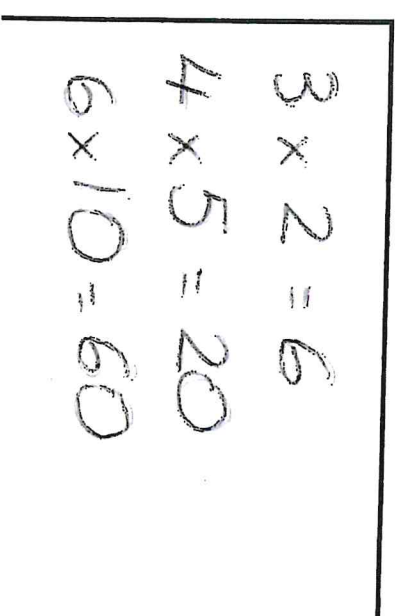
Multiplying by 2×10 and 5 (P)



Vocabulary

- array
- groups of
- lots of
- times

Multiplying by 2×10 and 5 (A)



At this stage, it is important that the children develop their mental recall of these tables.

Stage 4 – Multiplication in a range of contexts

Children identify that multiplication is the same as repeated addition

Multiplication as an array (P)

$$3 \times 4 = 12$$


Written abstract (A)

$$3 \times 4 = 12$$

Vocabulary

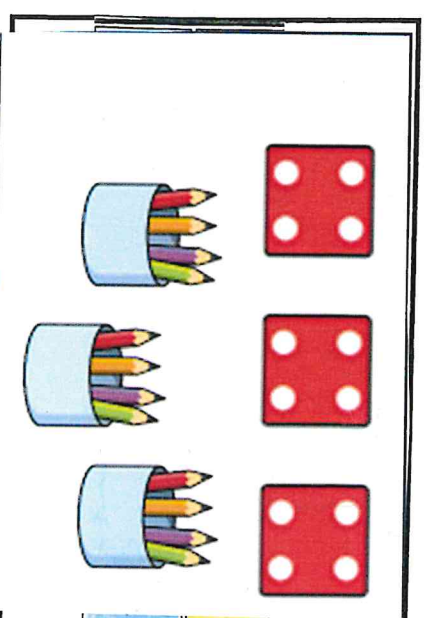
- array
- groups of
- lots of
- times
- repeated
- +++++
- bar model

Shown as repeated addition (P)

$$3 \times 4 = 12$$

$$4 + 4 + 4 = 12$$

(P)



Stage 5 – Multiplying 2 digit numbers by a single digit past 10.

Continue to drive times tables and mental methods

2 digit x 1 digit (past 10) (C)

$16 \times 3 = 48$

$$\begin{array}{r} 30 \\ + 18 \\ \hline \end{array}$$

2 digit x 1 digit (past 10) (A)

$16 \times 3 = 48$

x	10	6
3	30	18

$30 + 18 = 48$



Vocabulary

- array
- groups on
- lots of
- times
- repeated
- ++++
- multiply
- number
- line
- grid
- method



2 digit x 1 digit (past 10) (P)

$16 \times 3 = 48$

$30 + 18 = 48$

2 digit x 1 digit (past 10) (A)

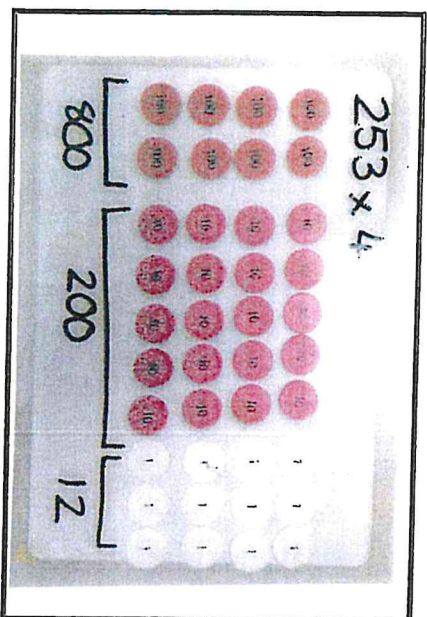
$16 \times 3 = 48$

$30 + 18 = 48$



Stage 6 – 3 digit x 1 digit (All tables should be known)

3 digit x 1 digit (C)



3 digit x 1 digit (A)

$$253 \times 4 = 1012$$

$$\begin{array}{r} 253 \\ \times 4 \\ \hline 1012 \end{array}$$

Vocabulary

- lots of
- multiply
- product
- grid
- method
- regroup
- column

3 digit - 1 digit (P)

$$253 \times 4 = 1012$$

x	200	50	3
4	800	200	12

3 digit x 1 digit (A)

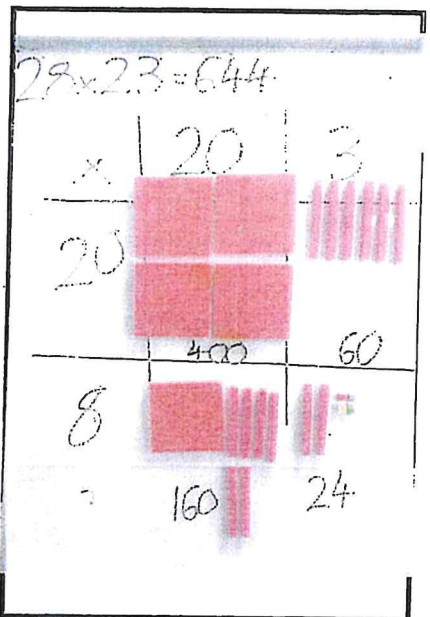
$$253 \times 4 = 1012$$

x	200	50	3
4	800	200	12

800 + 200 + 12 = 1012

Stage 7 – 2 digit x 2 digit

2 digit x 2 digit (C)



2 digit x 2 digit (A)

$$\begin{array}{r} 28 \\ \times 23 \\ \hline 84 \\ + 560 \\ \hline 644 \end{array}$$

- Vocabulary
- multiply
 - grid
 - method
 - product of

2 digit x 2 digit (grid method) (A)

$$\begin{array}{r} 28 \times 23 = \\ \textcircled{1} \times \begin{array}{|c|c|} \hline 20 & 3 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 20 & 400 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 60 = 460 \\ \hline \end{array} \\ \begin{array}{|c|} \hline 8 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 160 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 24 = 184 \\ \hline \end{array} \\ \hline \textcircled{2} \begin{array}{r} 460 \\ + 184 \\ \hline 644 \end{array} \end{array}$$

2 digit x 2 digit – expanded (A)

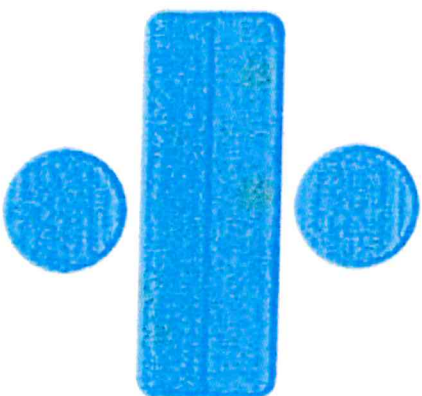
$$\begin{array}{r} 28 \\ \times 23 \\ \hline 24 \quad (8 \times 3) \\ 60 \quad (20 \times 3) \\ 160 \quad (8 \times 20) \\ + 400 \quad (20 \times 20) \\ \hline 644 \end{array}$$

division

how many times does?

share

remainders



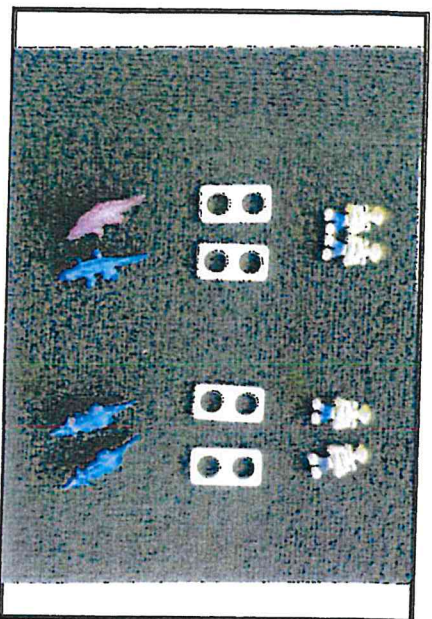
into groups of...

divide

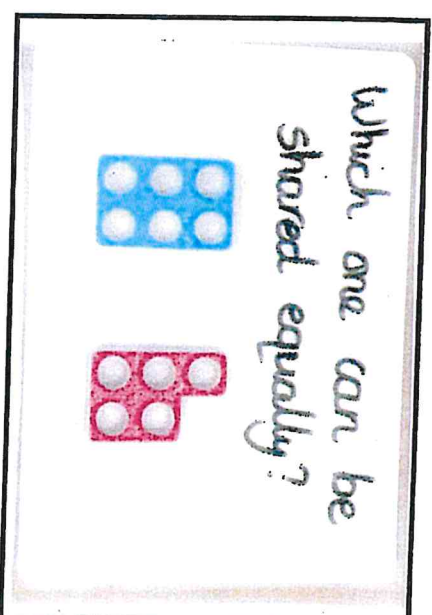
halving

Stage 1 – Introducing concept of sharing

Putting objects into pairs (C)



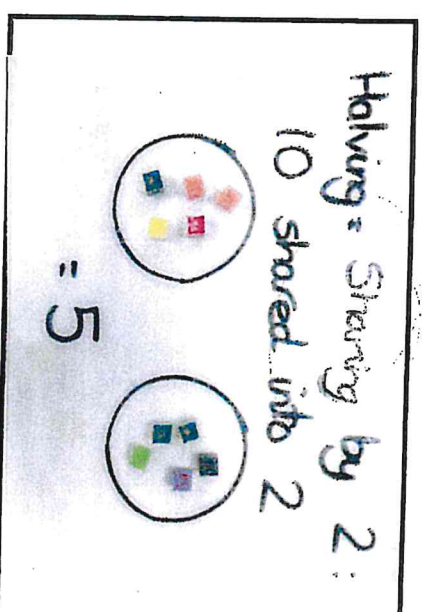
Putting objects into pairs (P)



Stage 1 targets:

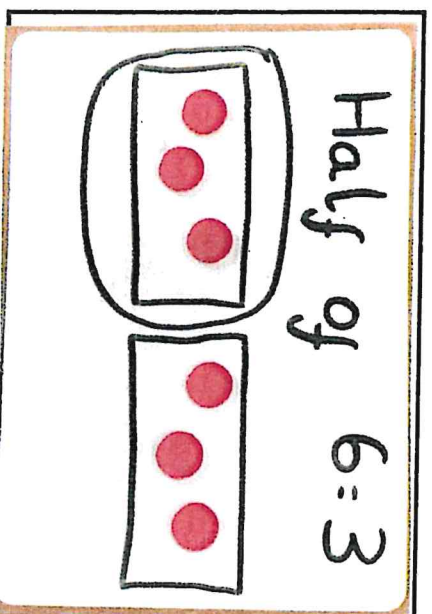
- Children need to understand the importance of sharing (toys, counters, symbols)
- Can they put a variety of objects into 2 equal groups?

Sharing and grouping by 2 (C)

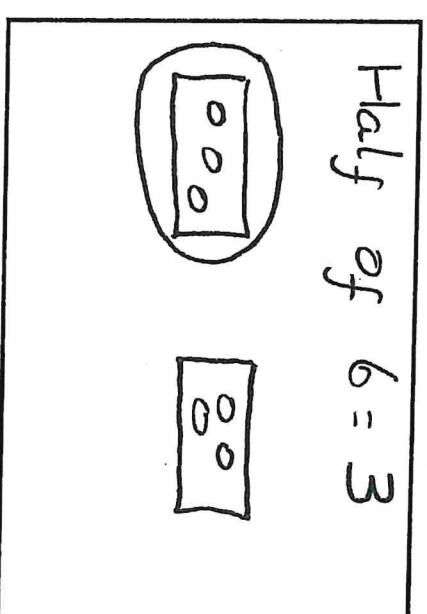


Stage 2 – Introduce concept of halving (2 equal groups)

Halving into 2 equal groups (C)



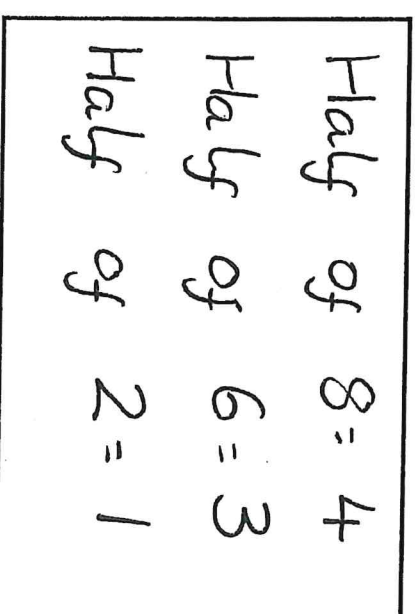
Halving into 2 equal groups (P)



Vocabulary

- Share by 2
- 2 equal groups
- halve

Halving (A)

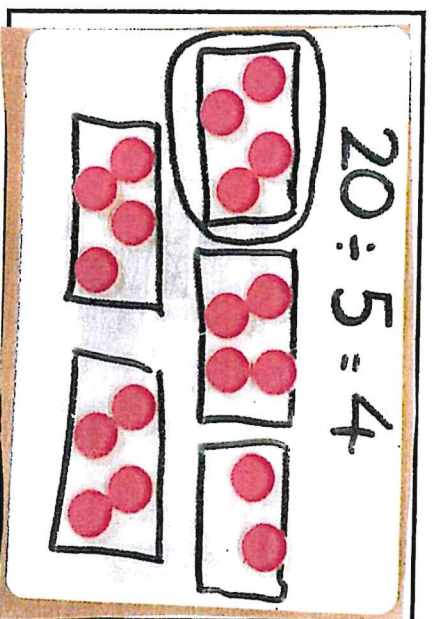


Stage 2 targets:

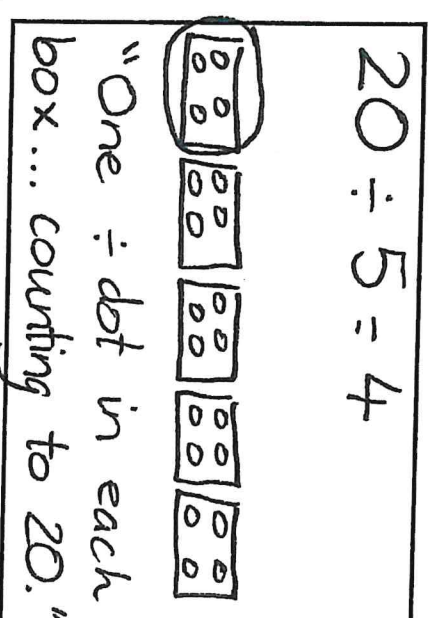
Important to address misconceptions with halving (has to be 2 equal groups).
Eventually, children need to halve numbers to 10 mentally.

Stage 3 – Introducing division symbol ($\div$), 10 and 5 (C) and division dots

Dividing by 2, 10 and 5 (C)



Dividing by 2, 10 and 5 (P)



Stage 3 targets:

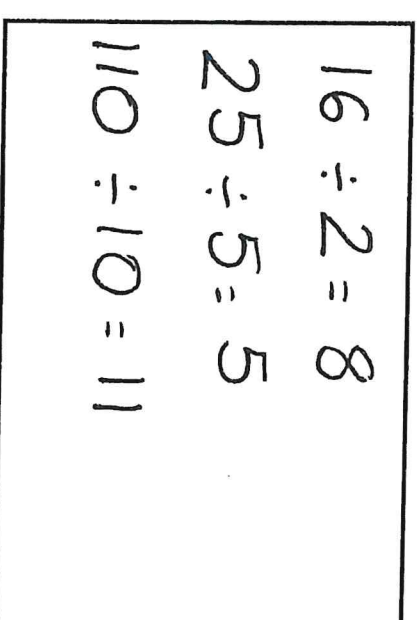
Children begin to spot relationship with Multiplication.

E.g. $35 \div 5 = 7$
 $7 \times 5 = 35$

Vocabulary

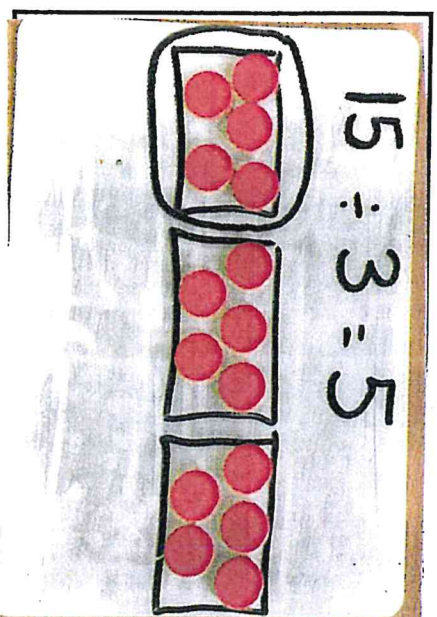
- share by
- equal
- groups of
- halve
- divide
- inverse
- division
- dots

Dividing by 2, 10 and 5 (A)



Stage 4 – Dividing by other numbers within the times tables

Dividing numbers (C)



Introducing remainders (C, P, A)

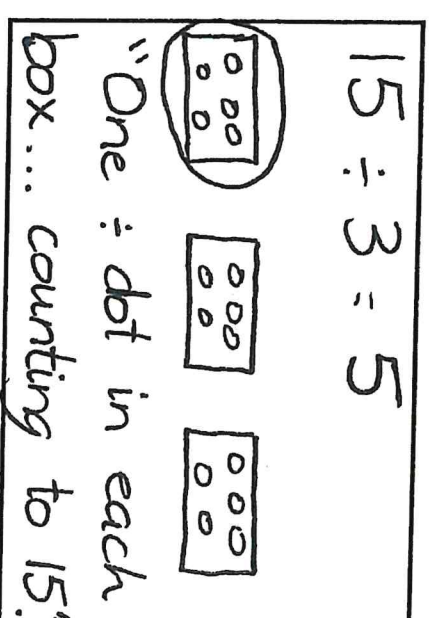


Vocabulary

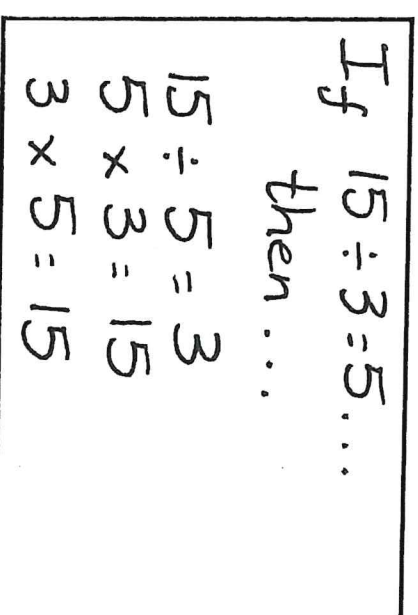
- share by
- equal
- groups of
- divide
- inverse
- remainders
- division
- dots



Dividing numbers (P)

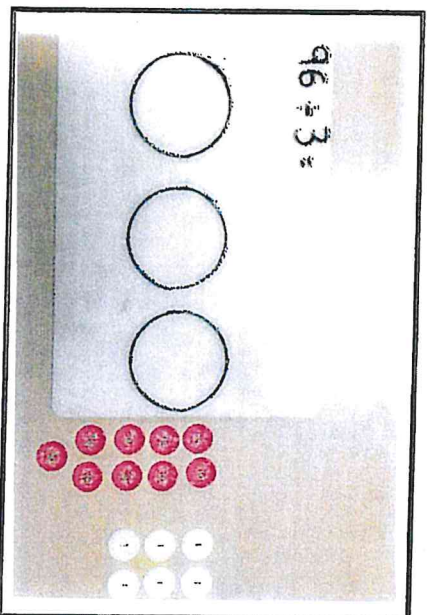


Dividing numbers (A)

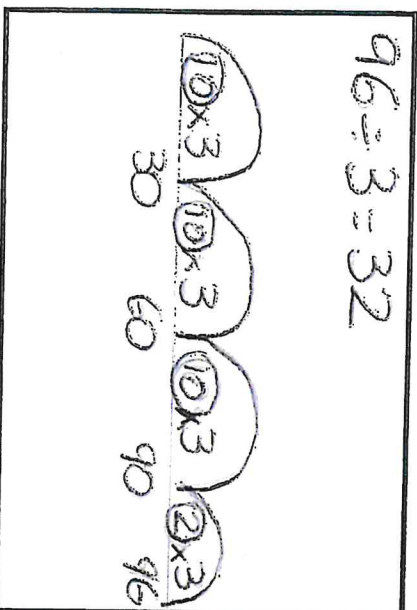


Stage 5 – Dividing past times tables (96 ÷ 3 =)

Dividing past times tables (C)



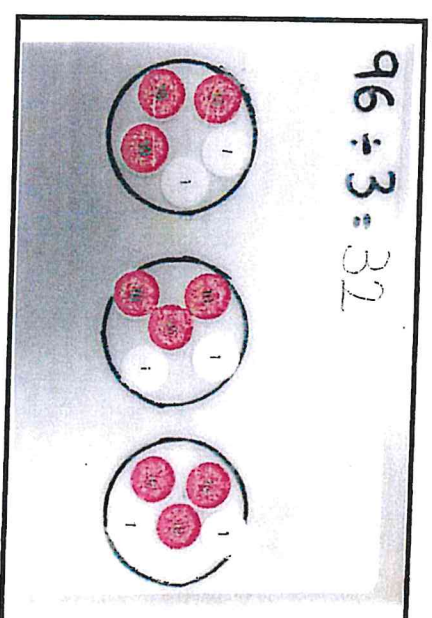
Dividing past times tables (A)



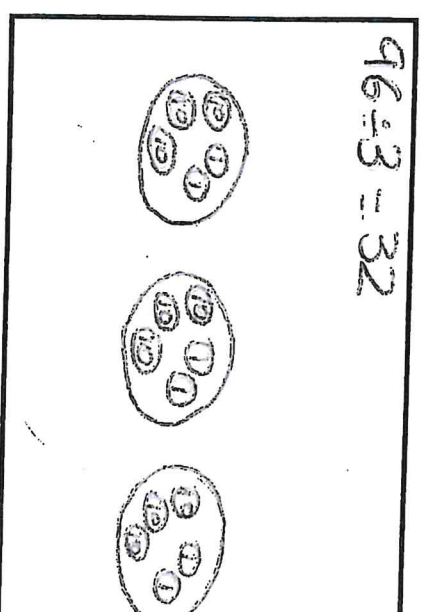
Vocabulary

- groups of
- divide
- share
- chunk
- use multiplication knowledge

Dividing past times tables (C)

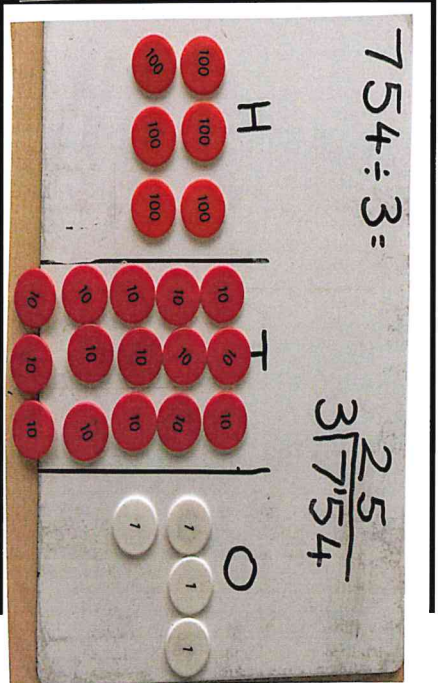
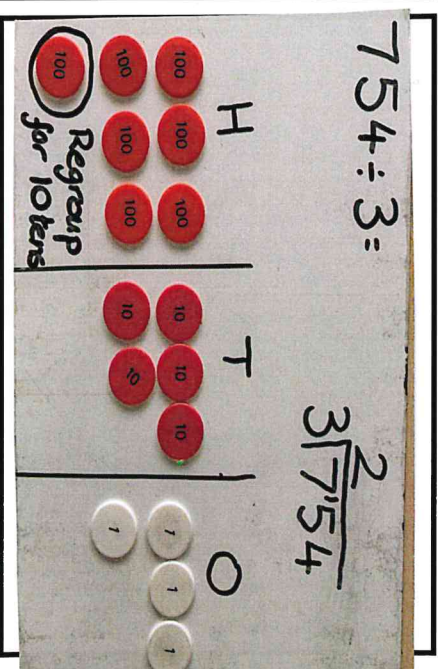
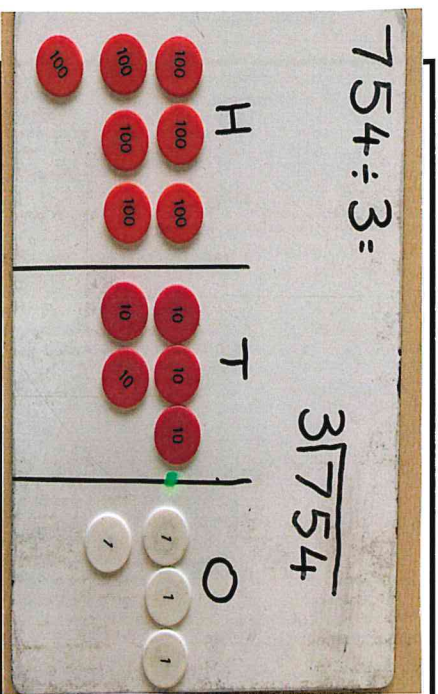


Dividing past times tables (P)

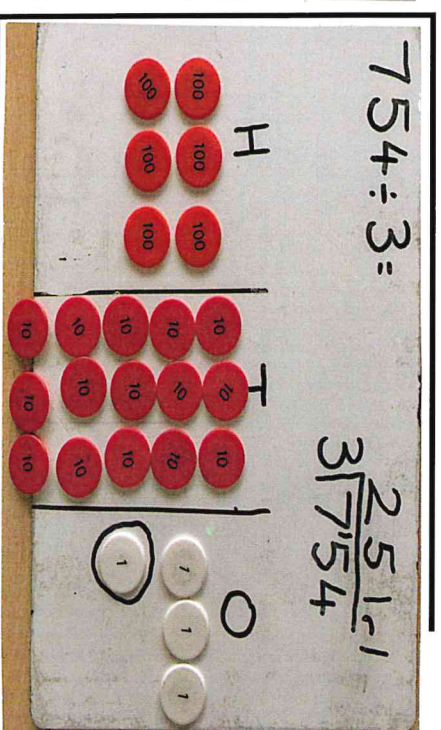
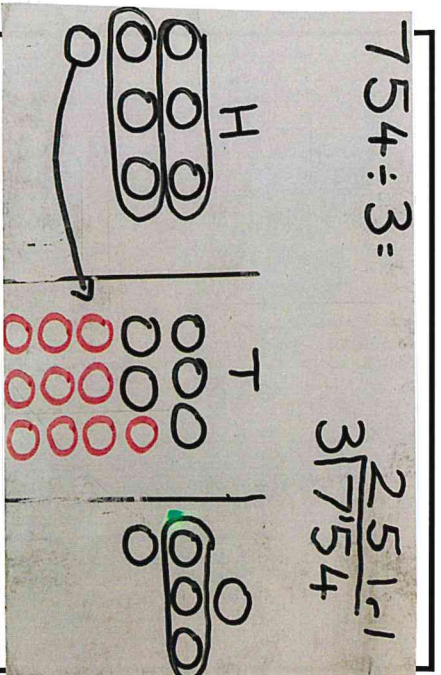
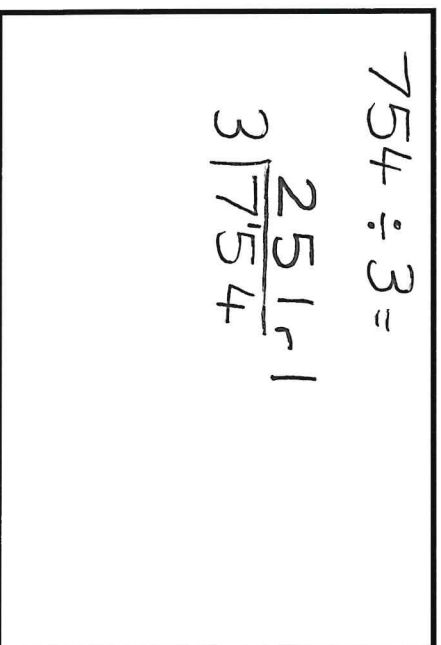


Stage 6 – 3 digit ÷ 1 digit (with regrouping e.g. $754 \div 3$)

3 digit ÷ 1 digit (C)



3 digit ÷ 1 digit (A)



Stage 7 – 4 digit ÷ 2 digit and converting remainders

4 digit ÷ 2 digit (A)

$$8657 \div 12 = 721 \text{ r } 5$$

$$\begin{array}{r} 0721 \text{ r } 5 \\ 12 \overline{) 8657} \end{array}$$

4 digit ÷ 2 digit (converting to fractions) (A)

$$9210 \div 12 = 767 \frac{1}{2}$$

$$\begin{array}{r} 0767 \text{ r } 6 \\ 12 \overline{) 9210} \end{array}$$

4 digit ÷ 2 digit (converting to decimals) (A)

$$4084 \div 16 = 255.25$$

$$\begin{array}{r} 0255 \text{ r } 4 \\ 16 \overline{) 4084} \end{array} = 0.25$$